An Unfair “Pool”?

A Statistical Sleuthing of the 2013 FINA Swimming World Championships

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Table of Contents:

Introduction:

During the 2013 FINA World Swimming Championships in Barcelona, there was much controversy as to whether the temporary Myrtha Pool was truly a “fair” pool. As the competition proceeded, many spectators started noticing unusual results, especially in the 50m events. The swimmers who were swimming closer to lane 8 were more likely to finish in the top 4, an unusual coincidence as the fastest seeded swimmers are placed in the middle of the pool (lanes 3,4,5,6) and in most competitions are the ones performing the best.

Moreover, for the longer events, athletes competing closer to the two sides of the pool noticed a large difference in their splits between odd and even 50s, even though almost no swimmers adopt that type of strategy. These strange results suggest that there may have been some type of current in the pool that mainly impacted the swimmers on the outsides of the pool, as lane 8 and lane 1 often had “flipped” relative 50m splits. Among the swimming community, there was much discussion theorizing the cause of this phenomenon.

In this study, we would like to add a quantitative statistical analysis to answer the question many competitors, spectators, and coaches had: Was there really a current in the pool at the 2013 FINA World Swimming Championships?

Methods:

1. Data Compilation and Organization

For our data, we accessed public records of the results from the 2013 FINA World Championships and the 2011 FINA World Championships via Omega Timing[1]. For our tests, we wanted to examine the 400m Freestyle, 1500m Freestyle, and all 4 sprint 50m events for both men and women. We decided to omit the 100m and 200m events from our analysis since we believed that there were too many confounding variables and variability within the the results to have reliable data. This is because there are many different strategies to swimming the 100m and 200m events, and we would only have 1 or 2 ratios for each swimmer. We also omitted the 800m freestyle from our analysis because we noticed that by testing the 400m and the 1500m, we indirectly had inferences for the 800m since the 400m had more variability while the 1500m had less variability. Thus, if both the 400m and 1500m events showed statistically significant results, then the 800m would also most likely result in statistically significant results.

With these raw data sets, we proceeded to create several calculations to assist us in our model. For the 400m and 1500m events, we created a "mean ratio" variable, which we defined to be "the mean of the ratios between odd and even 50s for each

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[1]
http://www.omegatiming.com/Competition?id=00010D0201FFFFFFFFFFFFFFFFFFFFFF&sport=AQ&year=2013
http://www.omegatiming.com/Competition?id=00010B0D00FFFFFFFFFFFFFFFFFFFFFF&sport=AQ&year=2011

individual swimmer". For example, in the 1500m Freestyle each swimmer has 15 ratios, so the "mean ratio" is the average of those 15 values. In our simple linear regression models for the distance freestyle events, we had "mean ratio" as our response variable and "lane" (whichever lane the individual swam in) as our predictor variable. For the 50m events, we created a "sum" variable, defined to be "the sum of the lanes of the top four swimmers in each championship final". This allowed us to create test statistics that allowed us to compare our two groups using hypothesis testing methods such as the Wilcoxon Rank-Sum Test.

2. Model Assumptions

Before we were able to proceed with any linear regression or t-tests, we had to make sure that our models didn't violate any of the assumptions needed to perform the aforementioned analyses. These assumptions are: normality, constant variance, linearity, and independence of observations.

For the 400m and 1500m events, we first plotted a histogram of our response variable (mean ratio) to evaluate whether or not we needed to transform the variable to adjust for any skewedness. We also plotted the Normal Q-Q Plot to assist us in this visualization of normality. Second, we plotted the residuals of the data to check for constant variance and linearity. Lastly, we can assume all the observations are independent because each individual swims their own race. In other words, no swimmer can swim in two lanes at the same time. One may argue that swimmers may be drafting off one another or be racing head-to-head influencing their opponent's race but for this analysis, we will assume that these effects are negligible in determining the splits and final results of the observations.

For the 50m events, we only performed a Wilcoxon Rank-Sum which doesn't depend on any underlying distribution since it's a non-parametric test hence we only needed to check for independence. A single observation was the sum of the lanes of the top 4 individuals in each championship final, so it is fair to assume that the results from one race will not affect the results from a different race.

3. Statistical Tests

We had three parts to the statistical tests that we performed. First, we ran simple regression models for the distance freestyle events to investigate if there was any relationship between lane and mean ratio. Second, we ran a series of unpooled two-sample tests comparing the lanes on opposite sides of the pool to help give us a more thorough understanding of the results from the linear regressions. Last, we performed a WIlcoxon Rank-Sum Test on the 50m events to see if there was any difference between the results from 2011 and 2013.

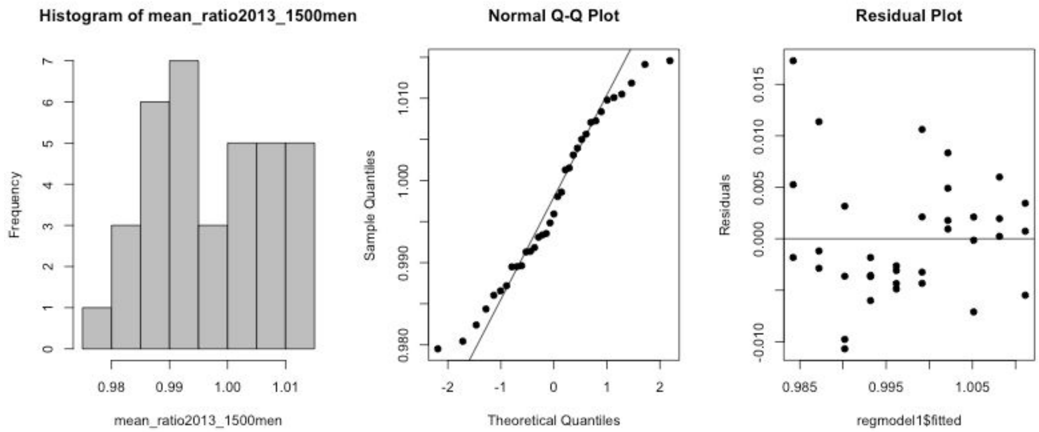
Results:

Assumptions (plots are below):

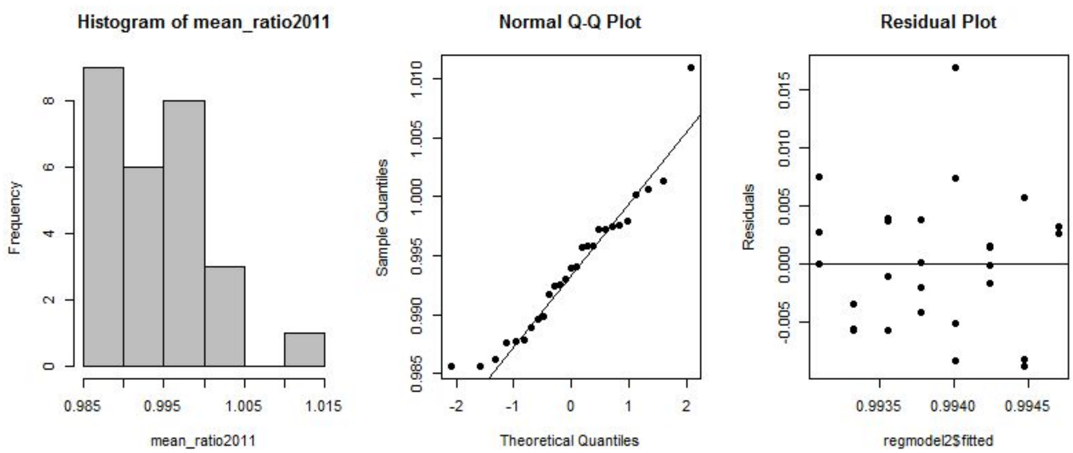
For the 1500m and 400m events for men and women, the assumptions needed for linear regression were mostly fulfilled. For those plots that showed slight violation of assumptions, transformations did little to fix the data. Moreover, many of these violations were caused by just one or two outliers and since our sample sizes were so small, the Central Limit Theorem wasn't in effect so these outliers tended to affect some of our normality and residual plots. We decided to proceed with caution for these events.

The men's 2013 1500m freestyle "mean ratio" histogram plot and Q-Q plot showed the data was approximately normal, while the residual plot showed linearity because the points were equally distributed along the x-axis and there's no curvature. The outliers in the plot may show a little non-constant variance, but the assumption isn't violated to the point where we can't proceed. The men's 2011 1500m freestyle "mean ratio" histogram plot and Q-Q plot showed the data was somewhat right skewed, but should the one outlier be removed the plot would be normal. We will proceed with caution. The residual plot demonstrated linearity and constant variance because there doesn't seem to be any curvature or fanning out of the points. The women's 2011 400m freestyle histogram plot showed one low outlier, but otherwise was normal. This event had constant variance and linearity. For the rest of the 400m and 1500m events, none of the assumptions seemed to be violated heavily so we were able to proceed with linear regression and t-tests.

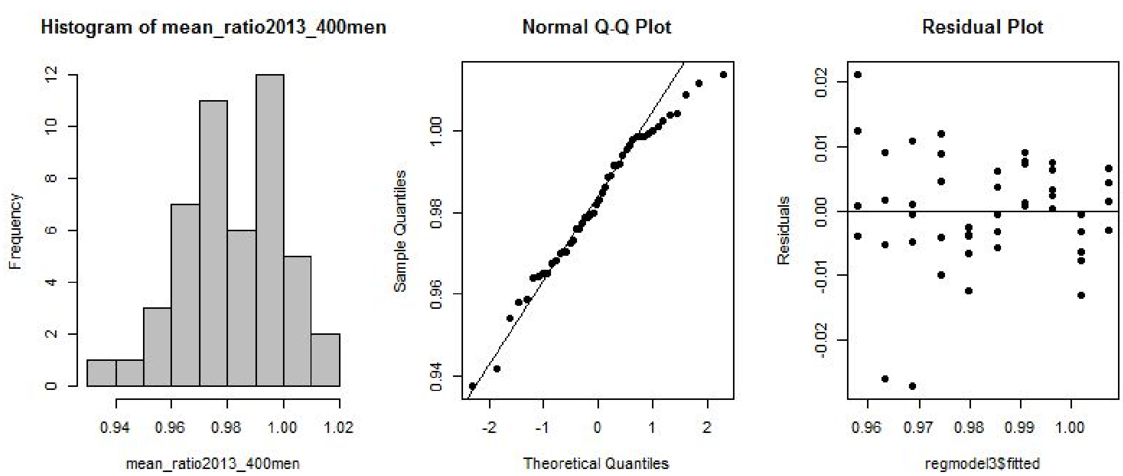
2013 Men 1500m Freestyle



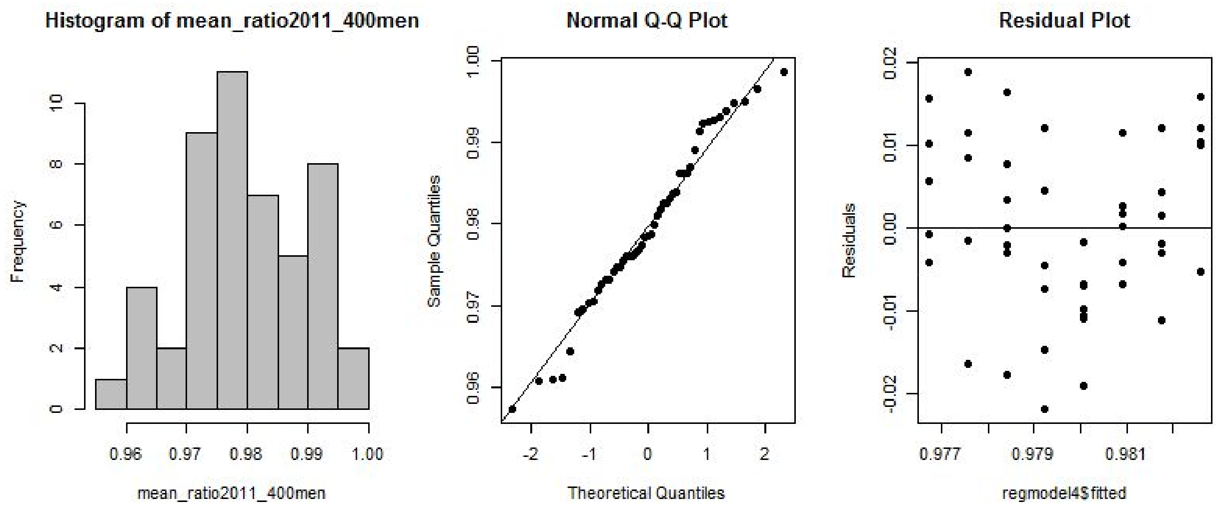
2011 Men 1500m Freestyle



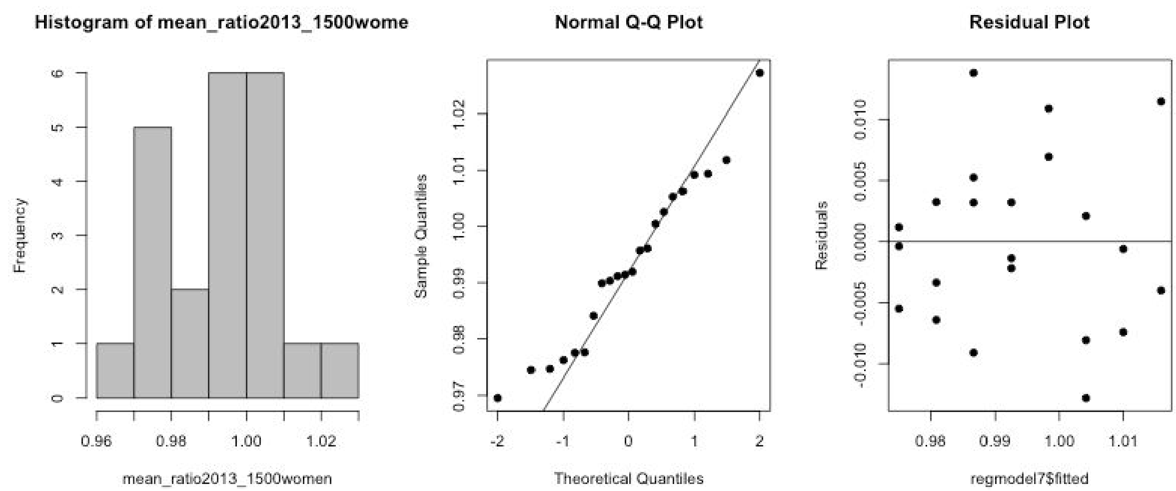
2013 Men 400m Freestyle



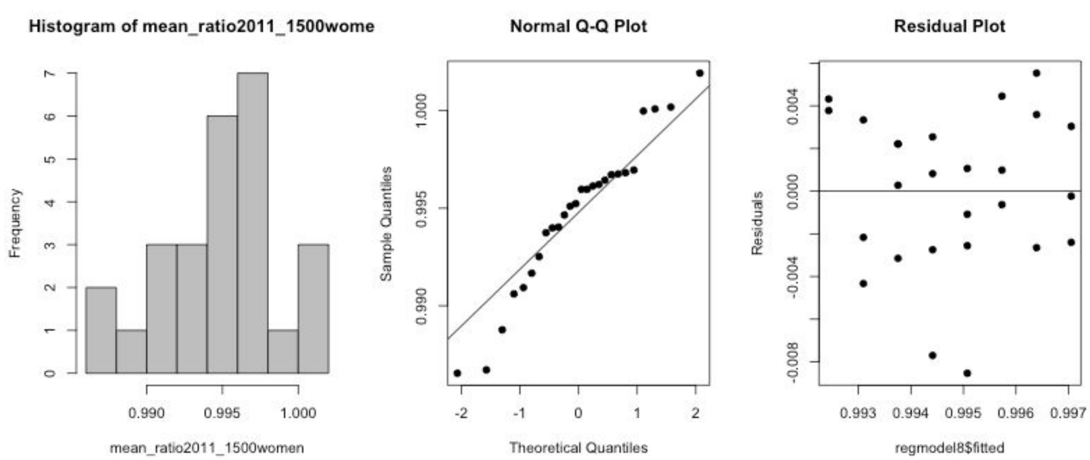
2011 Men 400m Freestyle



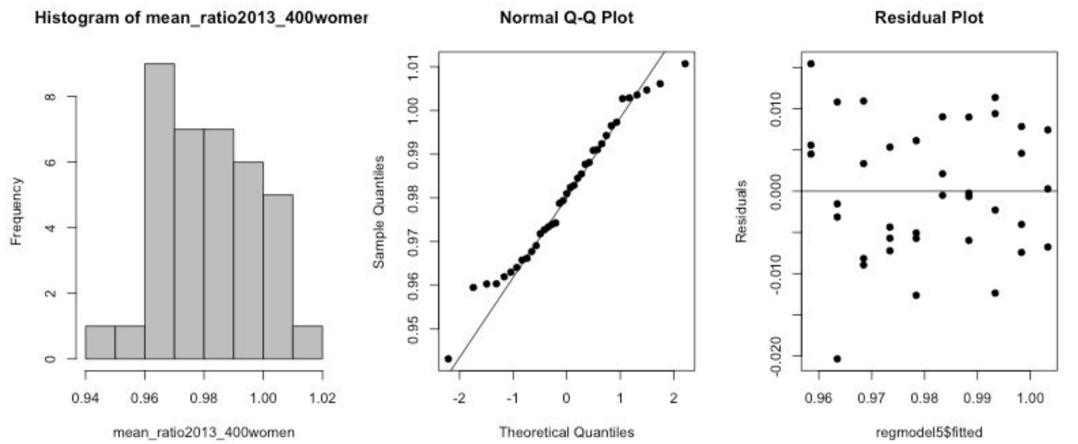
2013 Women 1500m Freestyle



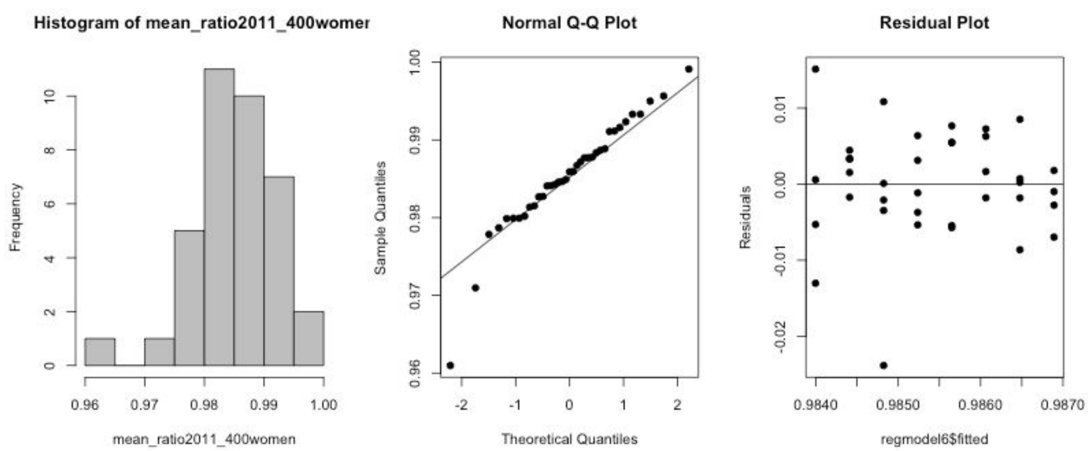
2011 Women 1500m Freestyle



2013 Women 400m Freestyle



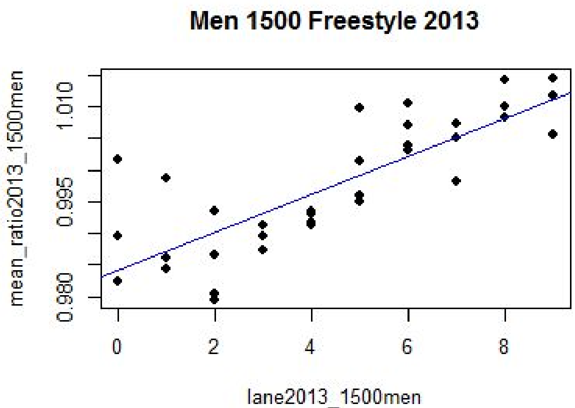
2011 Women 400m Freestyle:

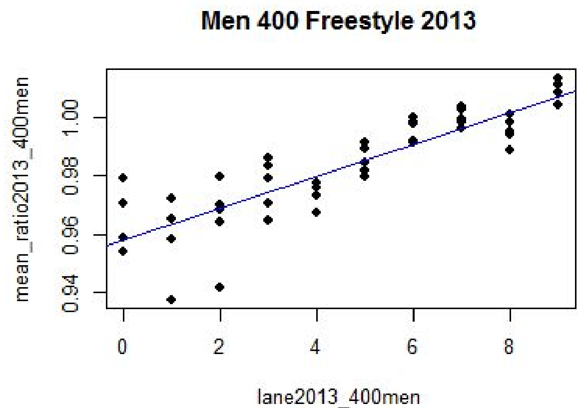


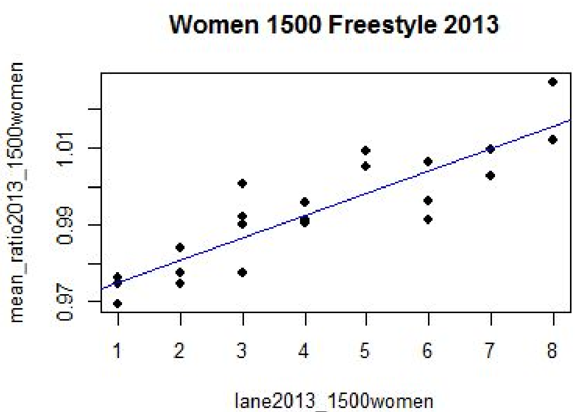
1. Simple Linear Regression Models:

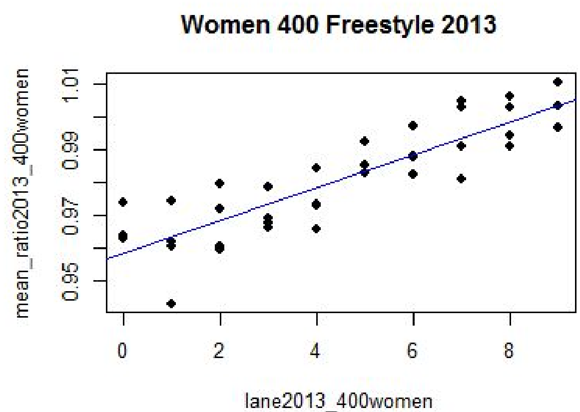
From Figures 1 through 8 below we can see the results for each of the regression models with “mean ratio” as the response variable and “lane” as the predictor variable.

2013, Test Group (400m and 1500m Freestyle)





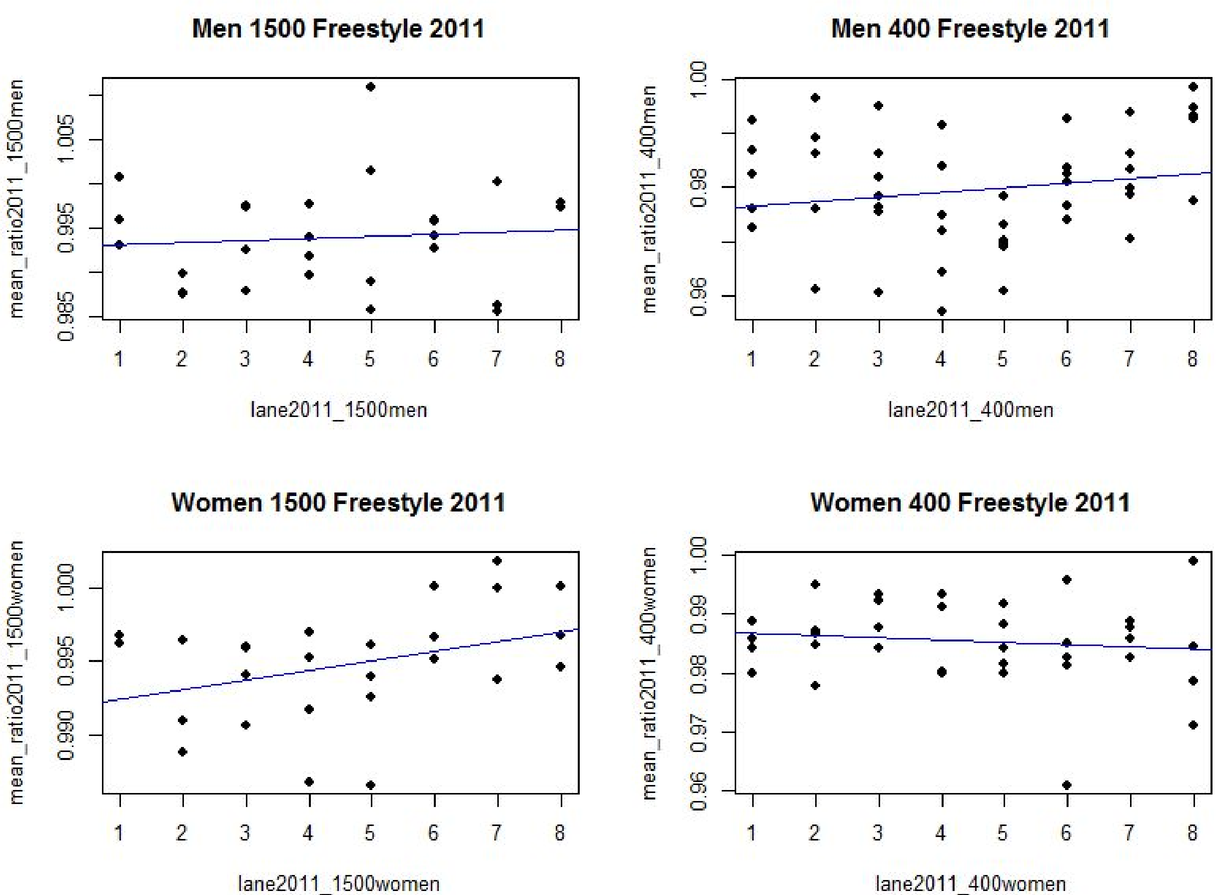




When looking at the results for each of the 2013 events we notice a similar trend which is that the regression lines clearly have positive slopes. This suggests that there is in fact a relationship between “lane” and “mean ratio”. This can be interpreted as the swimmers who swam closer to lane 0 were faster on odd 50s than even 50s, and swimmers who swam closer to lane 9 were slower on odd 50s than on even 50s. To further analyze this relationship, we can look at the summaries of these 4 regression models (Figures 1,3,5,7). From the summaries we can see that each of these 4 regression models have a positive slope estimate with a p-value much smaller than

0.05 (3.7e-09, 1.32e-15, 3.23e-12, 5.26e-08) which confirms that this relationship is statistically significant (this agrees with the visual model). Furthermore, to evaluate the strength and accuracy of these models we can look at their Adjusted R-squared values. The range for the 4 adjusted R-squared values is from 0.646 (Men 1500m Freestyle 2013) to 0.7689 (Women 1500m Freestyle 2013) which are relatively high adjusted R-squared values confirming that our regression models are accurate representations of the data points.

2011, Control Group (400m and 1500m Freestyle)



The results for the 2011 400m and 1500m events can be seen in the figures above. The main difference between these regression models and the ones for the 2013 events is the regression lines are much closer to being straight lines than upward sloping suggesting that there's no true relationship between "mean ratio" and "lane". This makes sense because the lane an individual swims in shouldn't affect how they swim their race. In other words, there shouldn't be a linear relationship between "mean ratio" and "lane". We can further explore this relationship by analyzing the summaries for these models. First, we notice that for all 4 models (Figures 2,4,6,8) the slope estimates have p-values greater than 0.05 (0.675, 0.214, 0.067, 0.435) which means that the slope of the regression line itself may not be an accurate representation of the data points. This could be for several reasons, the main one in this case seems to be high variance, which makes sense because every individual is unique and swims their own races. Again, we can evaluate the strength and accuracy of these models by looking at their Adjusted R-squared values. Here we see that the range for the 4 adjusted R-squared values is between -0.03255 to 0.09687 which are much lower values than we have for the 2013 models suggesting that these regression models may in fact be inaccurate models of the data. This makes sense because as we mentioned before, there shouldn't be a linear relationship between "mean ratio" and "lane" so it doesn't come to a surprise that trying to fit lines to these 4 scatterplots results in low adjusted R-squared values.

2. Unpooled 2-sample t-tests (Lane 1 vs Lane 8)

We also decided to perform unpooled 2-sample t-tests to compare the means of the "mean ratios" of lane 1 and lane 8 in each of the 400m and 1500m freestyle events. In these t-tests, the null hypothesis is that the means of the "mean ratios" are equal for lane 1 and lane 8 and the alternative hypothesis is that they are not equal. The results to these t-tests can be found in Figures 9 to 16. We chose to compare lane 1 and lane 8 because theoretically, if there was a current in the pool, there should be a significant difference between the means whereas if there's no current there should be no significant difference between the means. This is exactly what our results show.

For our control group (Figures 10, 12, 14, 16) we notice that all the p-values are greater than 0.10 meaning that even at the $\alpha = 0.10$ level we aren't able to reject the null hypothesis that the "mean ratios" are different for lanes 1 and lane 8 in 2011.

On the other hand, when looking at the t-test results for our test group (Figures 9, 11, 13, 15) we notice that 3 of the 4 tests result in p-values much smaller than 0.05 and the last test (Figure 13, Women 1500m Freestyle 2013) has a p-value of 0.08815. So here we are able to reject the null hypothesis at the $\alpha = 0.05$ significance level for 3 of the 4 groups and can reject the null hypothesis for all 4 groups at the $\alpha = 0.10$ significance level. Therefore there is sufficient evidence to conclude that for our test group (2013) there was truly a difference in the "mean ratios" between lanes 1 and 8.

3. Wilcoxon Rank-Sum Test (50m events)

```
> wilcox.exact(sum~year,alternative="two.sided")

        Exact Wilcoxon rank sum test

data:  sum by year
W = 54, p-value = 0.02005
alternative hypothesis: true mu is not equal to 0
```

```
nx = 8
ny = 8
totalrank = rank(sum, ties.method = "average")
rank_2011 = totalrank[year == 1]
tstat = sum(rank_2011)
var = var(totalrank)
rbar = (nx+ny+1)/2
rbar
zstat = (tstat-nx*rbar)/((sqrt(var))*sqrt(nx*ny/(nx+ny)))
zstat
pvalue=pnorm((zstat))*2
pvalue
```

```
> pvalue
[1] 0.01847584
```

For the 50m events we decided to perform a rank-sum test to compare the sum of the lanes of the top 4 swimmers in each championship final with the two groups being our control group (2011) and our test group (2013). Therefore each group had 8 observations (men and women, 4 stroke events). In this test our null hypothesis is that the sums of the the lanes of the top 4 swimmers for the 2011 50m events and for the 2013 50m events are equal. Also, we chose to sum the lanes of the top 4 swimmers and not only the top 3 because we wanted to make sure we took into account the fact that the pool has an even number of lanes otherwise there would be a bias towards lane 1 and 2. After performing the test by hand and in R, our tests resulted in p-values of 0.0185 and 0.02005 respectively (Figure 17). Thus we are able to reject the null hypothesis at the $\alpha = 0.05$ significance level and conclude that there's truly a difference between the sums of the lanes of the top 4 swimmers in the 50m events in 2011 and 2013. And using our results from the linear regressions, and t-tests comparing lanes 1 and 8, we can further conclude that the sum of the lanes of the top 4 swimmers in 2013 was greater than that in 2011 meaning that the swimmers swimming closer to lane 8 in 2013 were finishing in the top 4 in the 50m events more frequently than those swimming closer to lane 8 in 2011.

Discussion:

1. Implications

This study suggests that there was a significant association for difference in times depending on lane, leading to a preposition that there was a "current" in the pool. At the very least, the Barcelona pool at the 2013 FINA swimming championships is a case study for pool design in future events.

2. Limitations

Because our study was an observational study and not an experimental study, we have no external validity and thus cannot conclude any causal relationships. Therefore, we can't definitively say there was a current in the pool. We can, however, note that there was a strong association between the lane and time through our tests in this study. We also can't say that "all Myrtha pools" have this issue because we only sampled one out of many Myrtha Pools so our results aren't generalizable to the entire population of Myrtha Pools.

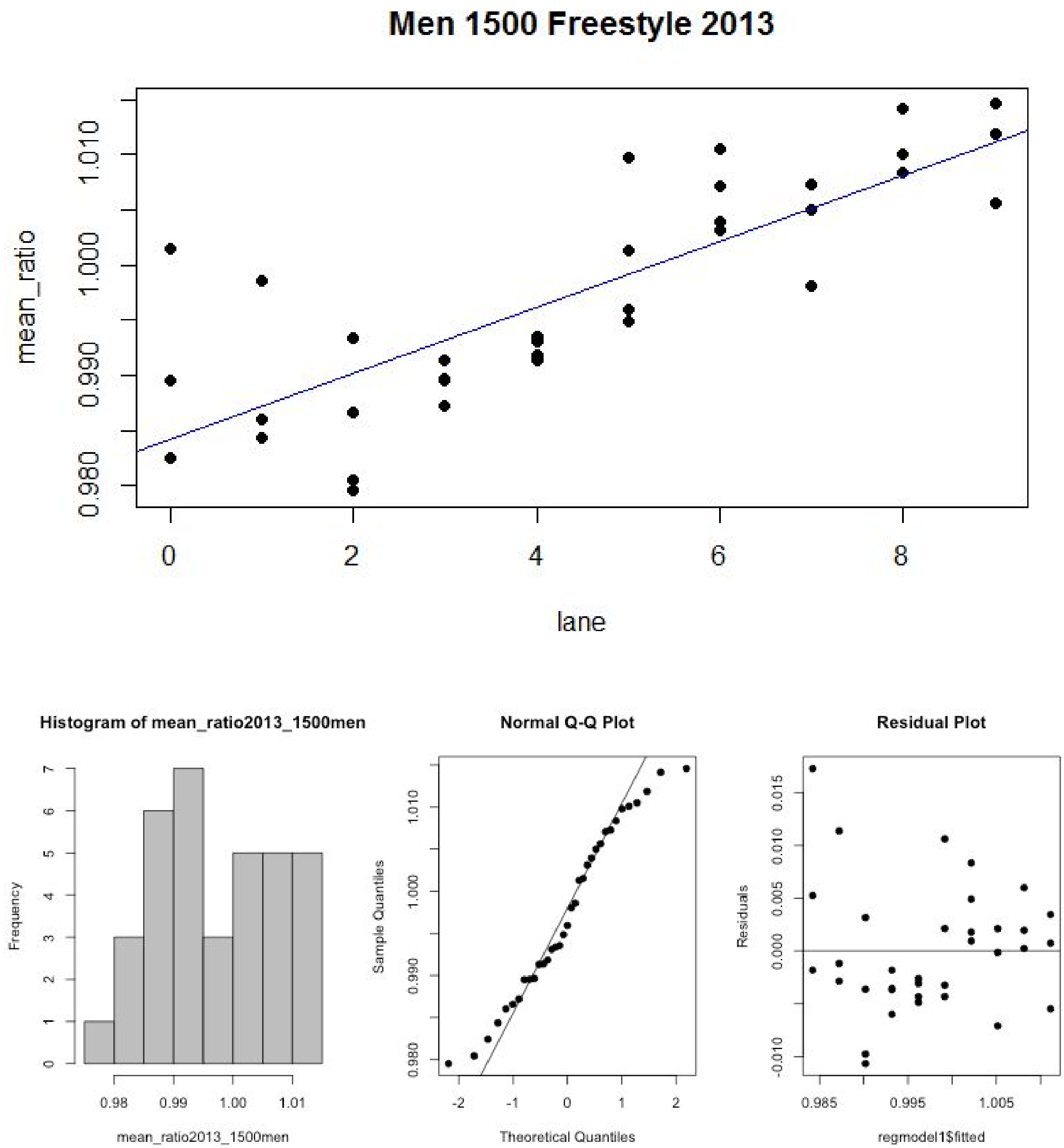
Another limitation to our study is that we did not include every event in the 2013 championships. Additionally, we do not include every year of the FINA World Championships — we do not know if this year is an anomaly and how it relates to other years besides 2011.

3. Next Steps

In future studies, we can include both more World Championships as well as more events within each championship. With more data points, we would be able to have a better understanding of how the 2013 World Championship compares to the rest of the championships. Although very difficult to implement, we could one day have experimentally designed competitions which would be different than regular competitions in that swimmers will be randomly selected to swim at this competition regardless of speed, and will also be randomly selected to swim in a lane through random number generation. With many competitions of this type, one may be able to conclude with certainty whether some pools really do have currents or not.

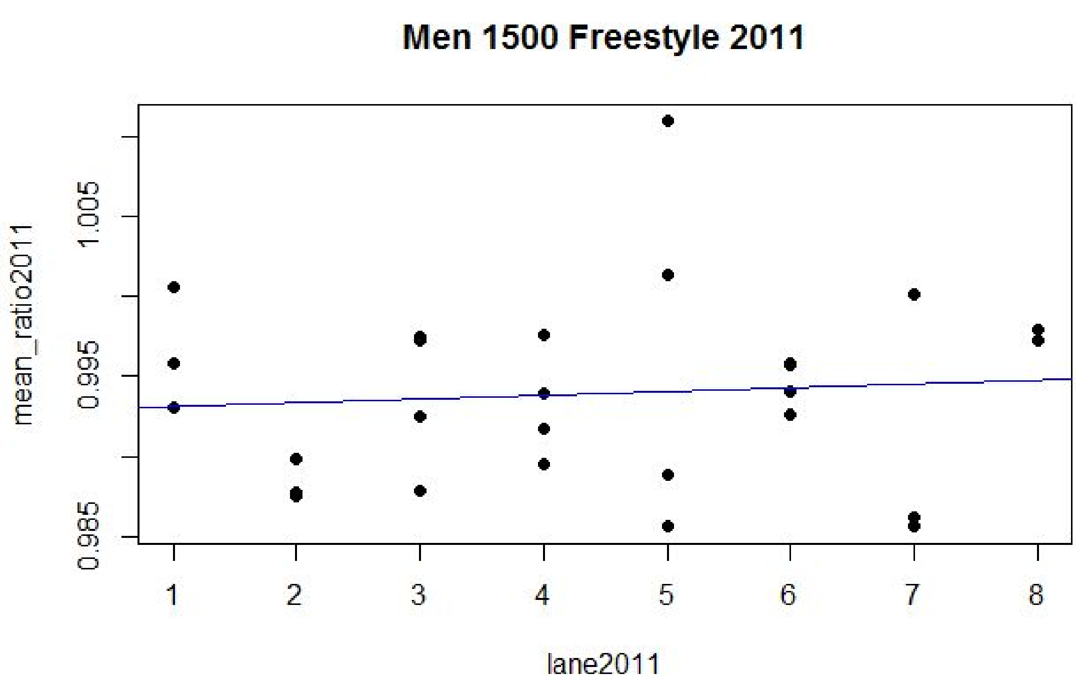
Figures

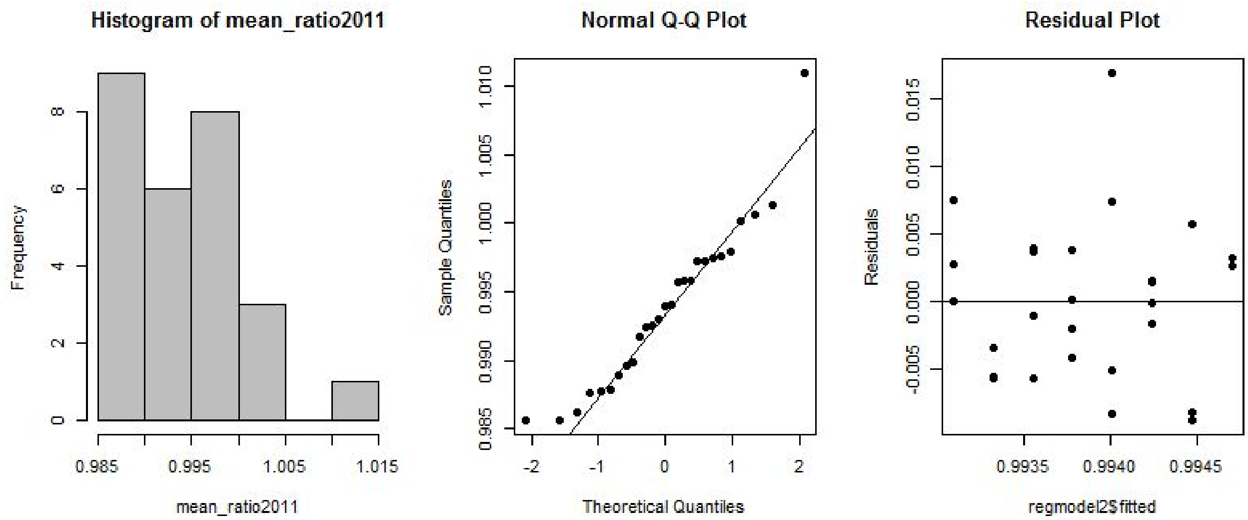
Figure 1:



```
> summary(regmodel1)

Call:
lm(formula = mean_ratio2013_1500men ~ lane2013_1500men, data = men1500_2013)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.010692 -0.003653 -0.001188  0.002642  0.017290 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)      0.9842308  0.0019550  503.43  < 2e-16 ***
lane2013_1500men 0.0029876  0.0003762    7.94  3.7e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.00605 on 33 degrees of freedom
Multiple R-squared:  0.6564,	Adjusted R-squared:  0.646 
F-statistic: 63.05 on 1 and 33 DF,  p-value: 3.705e-09
```

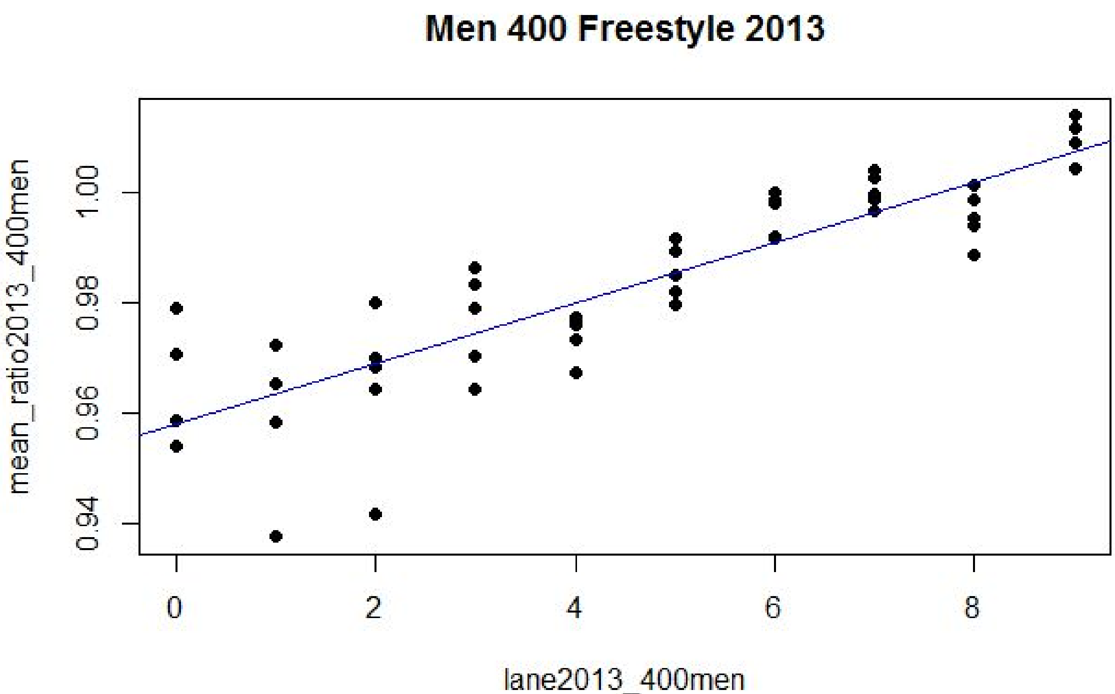
Figure 2:



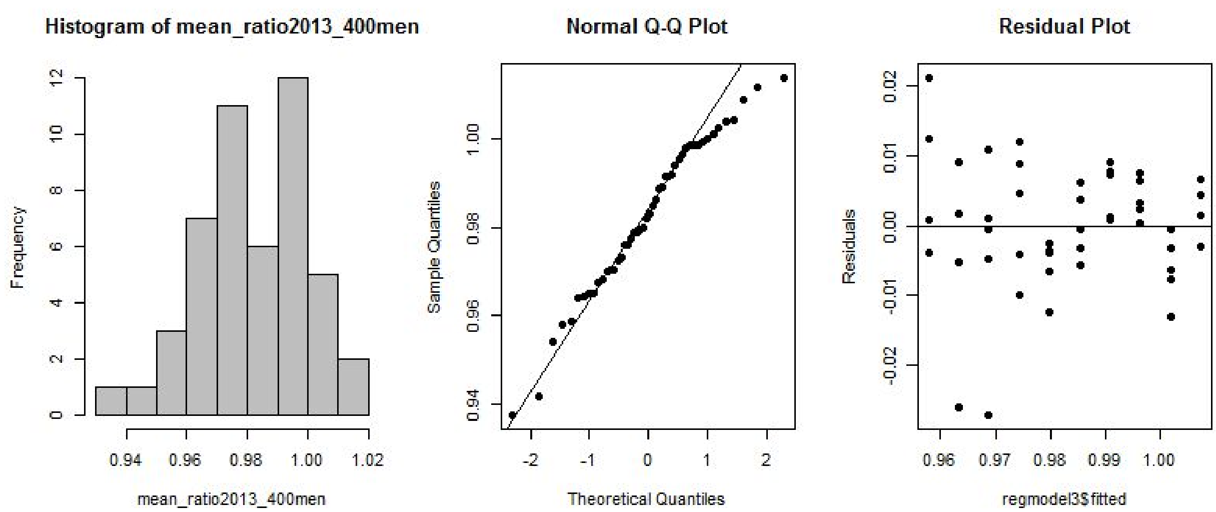



```
> summary(regmodel2)

Call:
lm(formula = mean_ratio2011_1500men ~ lane2011_1500men, data = men1500_2011)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0088736 -0.0046910 -0.0000383  0.0034230  0.0168967 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)      0.9928569  0.0026410 375.940   <2e-16 ***
lane2011_1500men 0.0002314  0.0005450   0.425    0.675    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.005928 on 25 degrees of freedom
Multiple R-squared:  0.007161,	Adjusted R-squared:  -0.03255 
F-statistic: 0.1803 on 1 and 25 DF,  p-value: 0.6747
```

Figure 3:



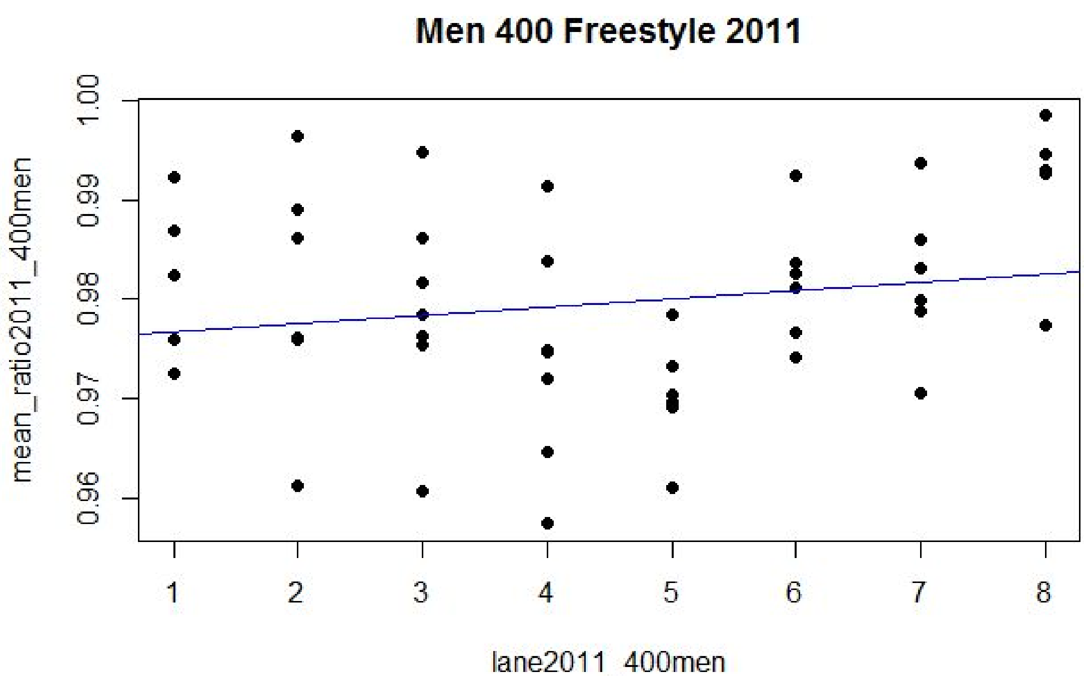
Men 400 Freestyle 2013
mean_ratio2013_400men
0.94
0.96
0.98
1.00
0
2
4
6
8
lane2013_400men

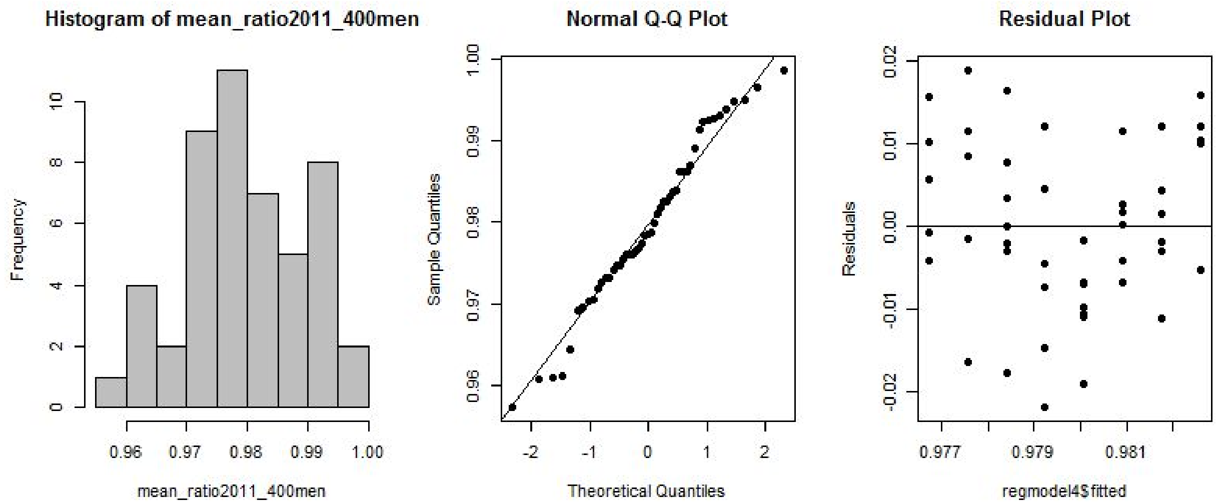


Histogram of mean_ratio2013_400men
Frequency
0
2
4
6
8
10
12
0.94
0.96
0.98
1.00
1.02
mean_ratio2013_400men
Normal Q-Q Plot
Sample Quantiles
0.94
0.96
0.98
1.00
-2
-1
0
1
2
Theoretical Quantiles
Residual Plot
Residuals
-0.02
-0.01
0.00
0.01
0.02
0.96
0.97
0.98
0.99
1.00
regmodel3$fitted

```
> summary(regmodel3)

Call:
lm(formula = mean_ratio2013_400men ~ lane2013_400men, data = men400_2013)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0271644 -0.0039411  0.0008046  0.0062405  0.0211468 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)     0.9578790  0.0024427  392.15  < 2e-16 ***
lane2013_400men 0.0054809  0.0004616   11.87 1.32e-15 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.008903 on 46 degrees of freedom
Multiple R-squared:  0.754,	Adjusted R-squared:  0.7486 
F-statistic:   141 on 1 and 46 DF,  p-value: 1.315e-15
```

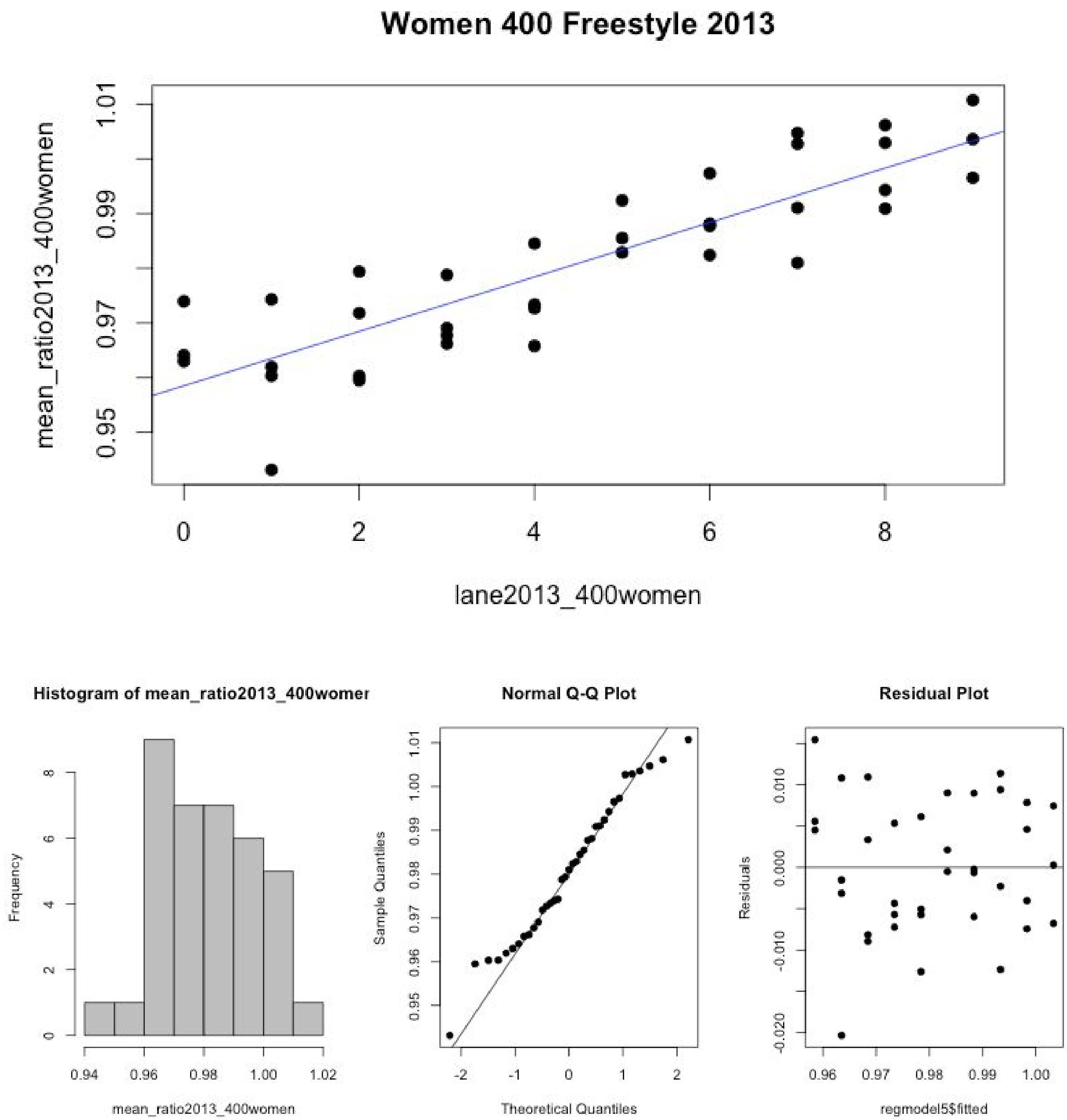
Figure 4:






```
> summary(regmodel4)

Call:
lm(formula = mean_ratio2011_400men ~ lane2011_400men, data = men400_2011)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.021922 -0.006801 -0.001453  0.008582  0.018889 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)     0.9758801  0.0033093 294.886   <2e-16 ***
lane2011_400men 0.0008398  0.0006662   1.261    0.214    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01011 on 47 degrees of freedom
Multiple R-squared:  0.0327,	Adjusted R-squared:  0.01212 
F-statistic: 1.589 on 1 and 47 DF,  p-value: 0.2137
```

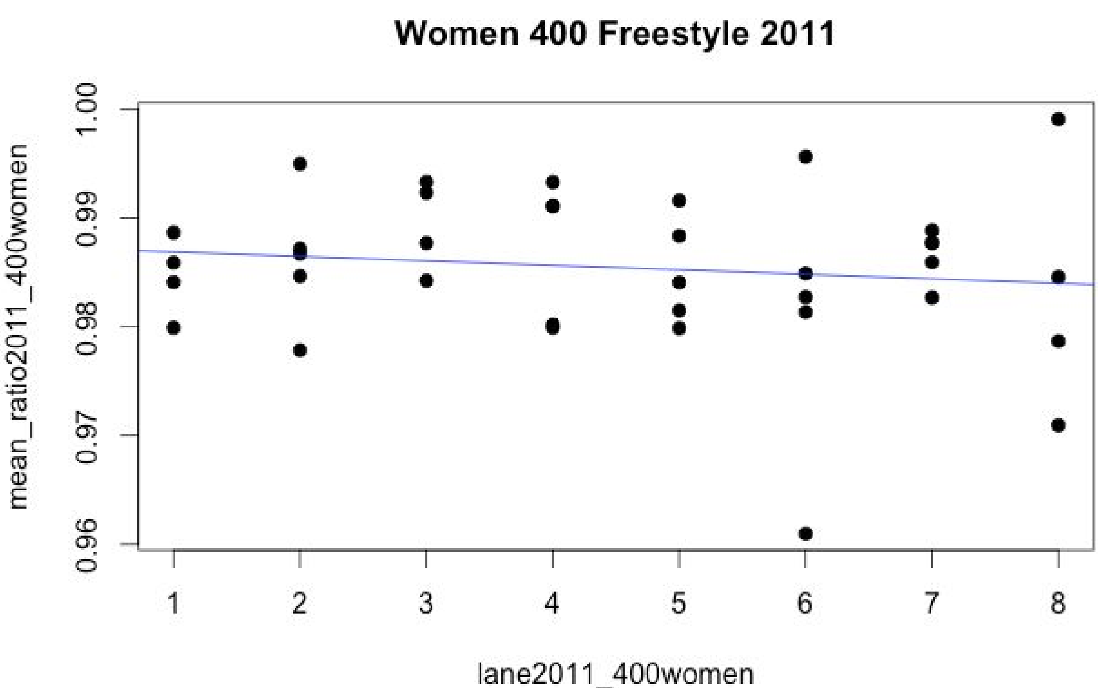
Figure 5:

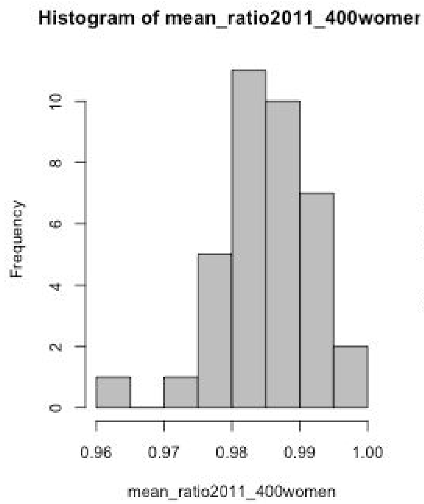


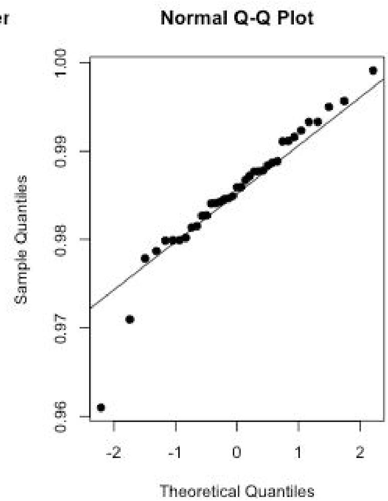
Women 400 Freestyle 2013
mean_ratio2013_400women
0.95
0.97
0.99
1.01
0
2
4
6
8
lane2013_400women
Histogram of mean_ratio2013_400womer
Frequency
mean_ratio2013_400women
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
Residual Plot
Residuals
regmodel5$fitted

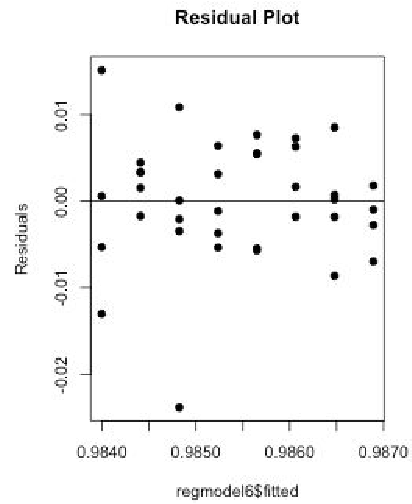
```
> summary(regmodel5)

Call:
lm(formula = mean_ratio2013_400women ~ lane2013_400women, data = women400_2013)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0203386 -0.0057264 -0.0005039  0.0061248  0.0154484 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)       0.9585030  0.0025364  377.89  < 2e-16 ***
lane2013_400women 0.0049765  0.0004798   10.37 3.23e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.00816 on 35 degrees of freedom
Multiple R-squared:  0.7545,	Adjusted R-squared:  0.7475 
F-statistic: 107.6 on 1 and 35 DF,  p-value: 3.232e-12
```

Figure 6:



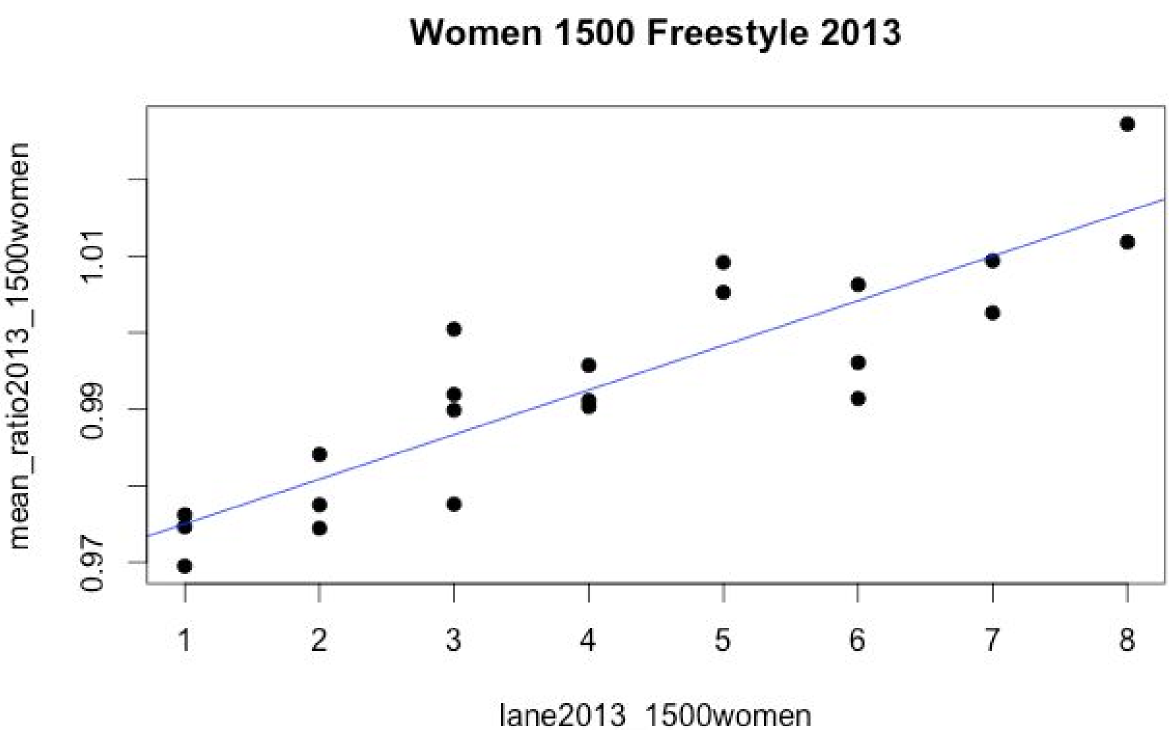




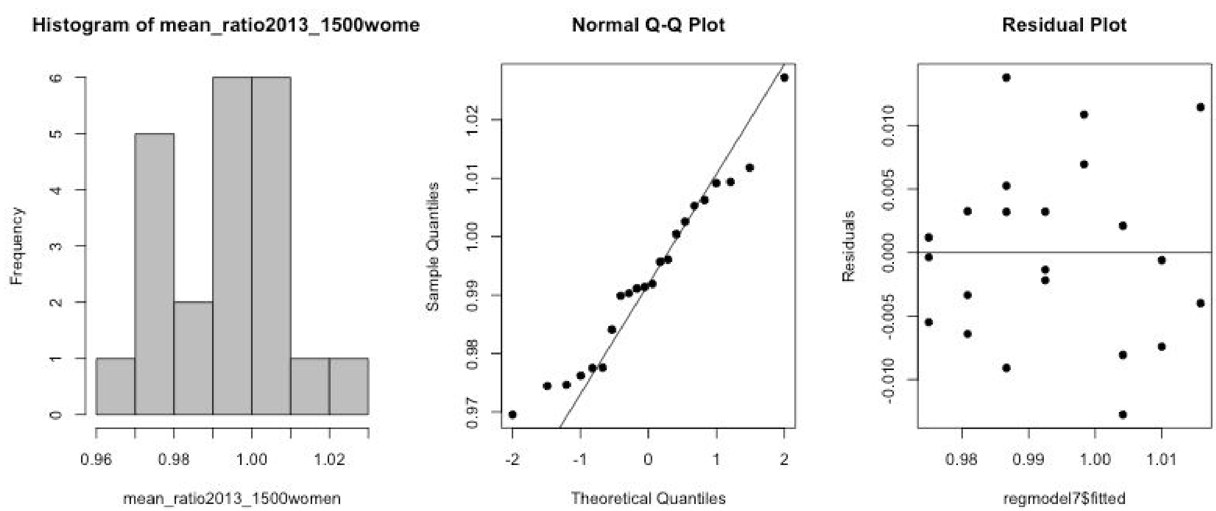



```
> summary(regmodel6)

Call:
lm(formula = mean_ratio2011_400women ~ lane2011_400women, data = women400_2011)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0238429 -0.0034623  0.0002385  0.0044485  0.0151192 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)        0.9873051  0.0026497  372.61   <2e-16 ***
lane2011_400women -0.0004137  0.0005240   -0.79    0.435    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.007093 on 35 degrees of freedom
Multiple R-squared:  0.0175,	Adjusted R-squared:  -0.01057 
F-statistic: 0.6233 on 1 and 35 DF,  p-value: 0.4351
```

Figure 7:



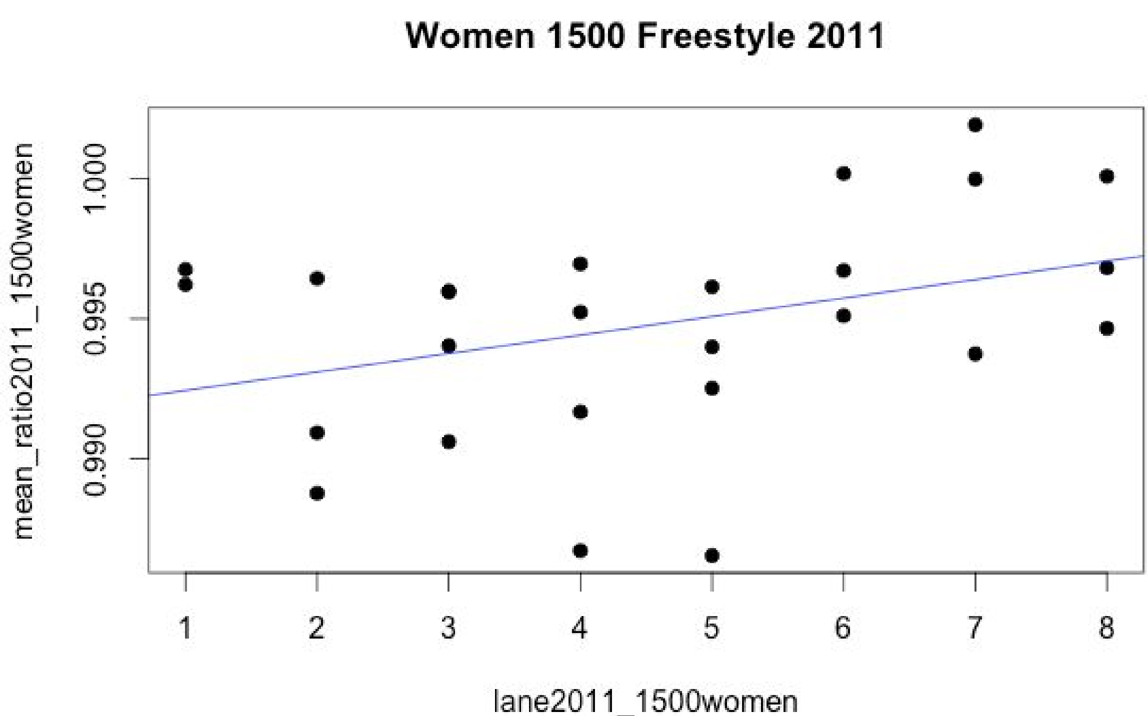
Women 1500 Freestyle 2013
mean_ratio2013_1500women
0.97
0.99
1.01
1
2
3
4
5
6
7
8
lane2013_1500women

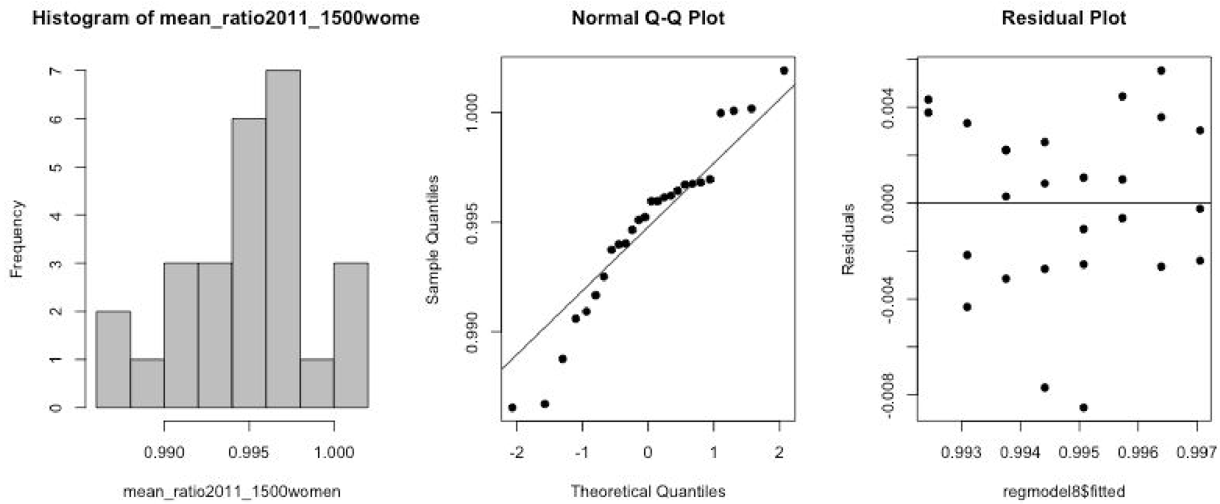


Histogram of mean_ratio2013_1500wome
Frequency
mean_ratio2013_1500women
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
Residual Plot
Residuals
regmodel7$fitted

```
> summary(regmodel7)

Call:
lm(formula = mean_ratio2013_1500women ~ lane2013_1500women, data = women1500_2013)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0127726 -0.0051214 -0.0004944  0.0032343  0.0137730 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)        0.9691931  0.0032449 298.681  < 2e-16 ***
lane2013_1500women 0.0058296  0.0006925   8.418 5.26e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.00715 on 20 degrees of freedom
Multiple R-squared:  0.7799,	Adjusted R-squared:  0.7689 
F-statistic: 70.86 on 1 and 20 DF,  p-value: 5.262e-08
```

Figure 8:






```
> summary(regmodel8)

Call:
lm(formula = mean_ratio2011_1500women ~ lane2011_1500women, data = women1500_2011)

Residuals:
       Min         1Q     Median         3Q        Max 
-0.0085348 -0.0025167  0.0005492  0.0029139  0.0055331 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)        0.9917759  0.0017328 572.343   <2e-16 ***
lane2011_1500women 0.0006594  0.0003437   1.919    0.067 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003707 on 24 degrees of freedom
Multiple R-squared:  0.133,	Adjusted R-squared:  0.09687 
F-statistic: 3.681 on 1 and 24 DF,  p-value: 0.06699
```

Figure 9:

Men 1500m Freestyle 2013

```
> t.test(mean_ratio2013_1500men[lane2013_1500men == 1],mean_ratio2013_1500men[lane2013_1
500men == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2013_1500men[lane2013_1500men == 1] and mean_ratio2013_1500men[lane2013
_1500men == 8]
t = -4.4141, df = 2.566, p-value = 0.02968
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.03806433 -0.00434267
sample estimates:
mean of x mean of y
0.9896563 1.0108598
```

Figure 10:

Men 1500m Freestyle 2011

```
> t.test(mean_ratio2011_1500men[lane2011_1500men == 1],mean_ratio2011_1500men[lane2011_1
500men == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2011_1500men[lane2011_1500men == 1] and mean_ratio2011_1500men[lane2011
_1500men == 8]
t = -0.4954, df = 2.079, p-value = 0.6677
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.010380225  0.008166297
sample estimates:
mean of x mean of y
0.9964841 0.9975911
```

Figure 11:

Men 400m Freestyle 2013

```
> t.test(mean_ratio2013_400men[lane2013_400men == 1],mean_ratio2013_400men[lane2013_400m
en == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2013_400men[lane2013_400men == 1] and mean_ratio2013_400men[lane2013_40
0men == 8]
t = -5.6546, df = 4.999, p-value = 0.002405
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.05230210 -0.01960891
sample estimates:
mean of x mean of y
0.9596307 0.9955862
```

Figure 12:

Men 400m Freestyle 2011

```
> t.test(mean_ratio2011_400men[lane2011_400men == 1],mean_ratio2011_400men[lane2011_400m
en == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2011_400men[lane2011_400men == 1] and mean_ratio2011_400men[lane2011_40
0men == 8]
t = -1.8118, df = 7.999, p-value = 0.1076
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.02098153  0.00251851
sample estimates:
mean of x mean of y
0.9820594 0.9912909
```

Figure 13:

Women 1500m Freestyle 2013

```
> t.test(mean_ratio2013_1500women[lane2013_1500women == 1],mean_ratio2013_1500women[lane
2013_1500women == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2013_1500women[lane2013_1500women == 1] and mean_ratio2013_1500women[la
ne2013_1500women == 8]
t = -5.7791, df = 1.138, p-value = 0.08815
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.12248004  0.03030032
sample estimates:
mean of x mean of y
0.9734566 1.0195464
```

Figure 14:

Women 1500m Freestyle 2011

```
> t.test(mean_ratio2011_1500women[lane2011_1500women == 1],mean_ratio2011_1500women[lane
2011_1500women == 8], alternative = "two.sided", data = men1500_2013)

	Welch Two Sample t-test

data:  mean_ratio2011_1500women[lane2011_1500women == 1] and mean_ratio2011_1500women[la
ne2011_1500women == 8]
t = -0.437, df = 2.116, p-value = 0.7027
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.007248263  0.005847262
sample estimates:
mean of x mean of y
0.9964863 0.9971868
```

Figure 15:

Women 400m Freestyle 2013

```
> t.test(mean_ratio2013_400women[lane2013_400women == 1],mean_ratio2013_400women[lane201
3_400women == 8], alternative = "two.sided", data = men1500_2013)

        Welch Two Sample t-test

data:  mean_ratio2013_400women[lane2013_400women == 1] and mean_ratio2013_400women[lane2
013_400women == 8]
t = -5.2657, df = 4.707, p-value = 0.00392
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.05785154 -0.01941483
sample estimates:
mean of x mean of y
0.9599287 0.9985619
```

Figure 16:

Women 400m Freestyle 2011

```
> t.test(mean_ratio2011_400women[lane2011_400women == 1],mean_ratio2011_400women[lane201
1_400women == 8], alternative = "two.sided", data = men1500_2013)

        Welch Two Sample t-test

data:  mean_ratio2011_400women[lane2011_400women == 1] and mean_ratio2011_400women[lane2
011_400women == 8]
t = 0.2119, df = 3.566, p-value = 0.8437
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.01683406  0.01947380
sample estimates:
mean of x mean of y
0.9846580 0.9833381
```

Figure 17:

$$Z = \frac{T - n_x R_{average}}{S_R \sqrt{\frac{n_x n_y}{n_x + n_y}}}$$

```
nx = 8
ny = 8
totalrank = rank(sum, ties.method = "average")
rank_2011 = totalrank[year == 1]
tstat = sum(rank_2011)
var = var(totalrank)
rbar = (nx+ny+1)/2
rbar
zstat = (tstat-nx*rbar)/((sqrt(var))*sqrt(nx*ny/(nx+ny)))
zstat
pvalue=pnorm((zstat))*2
pvalue
```

```
> pvalue
[1] 0.01847584
```

```
> wilcox.exact(sum~year,alternative="two.sided")

        Exact Wilcoxon rank sum test

data:  sum by year
W = 54, p-value = 0.02005
alternative hypothesis: true mu is not equal to 0
```

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